Transformations

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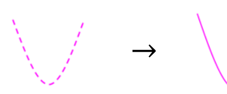
Single Transformations

Consider a graph/curve such as

Note: The curve has been drawn dashed since it will be use in the diagram below and there needs to be a distinction between the original curve (dashed) and transformed new curve (not dashed)

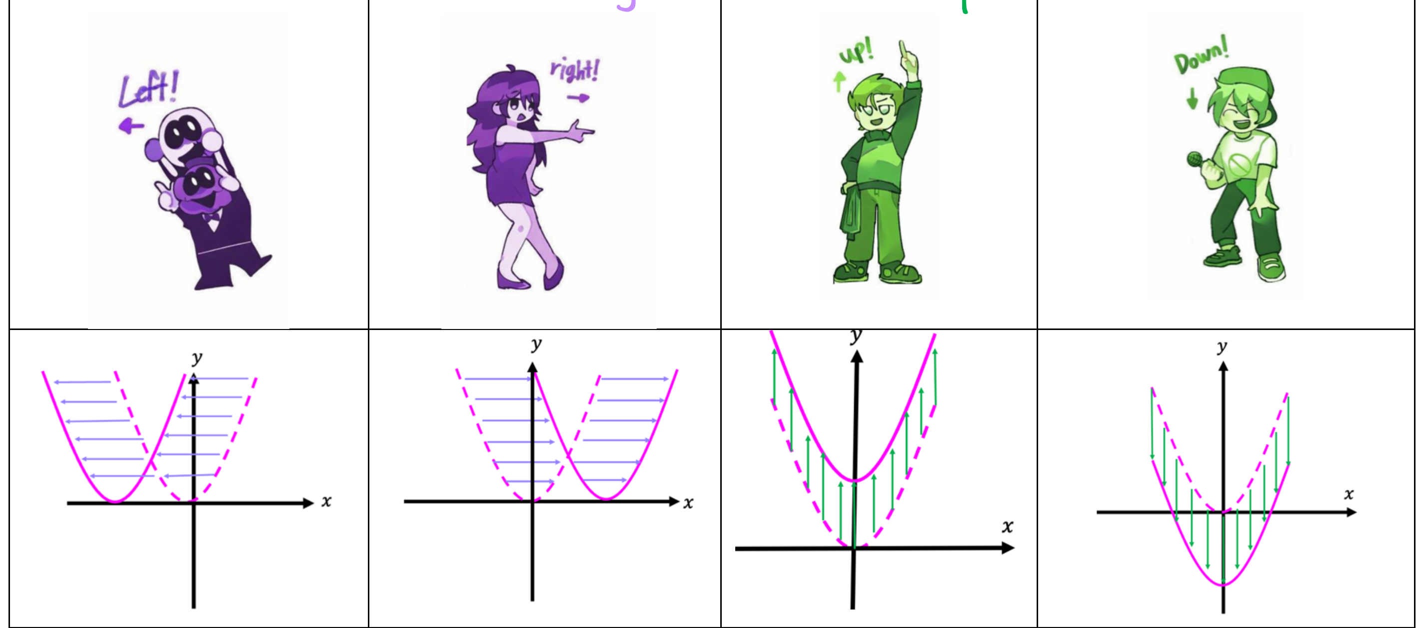


Lets give it a general name $f(x)$

We can do many things to this graph/curve

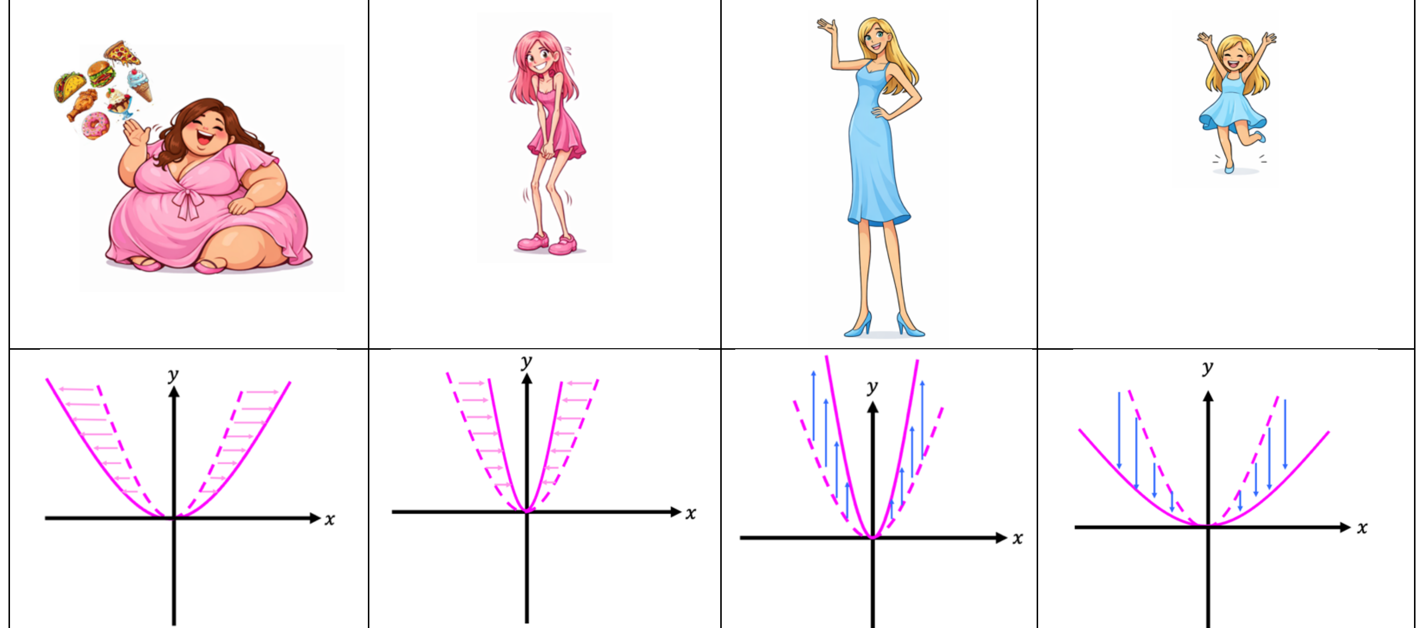
Translations (pick up and move)

Move It Left | Move It Right | Move It Up | Move It Down



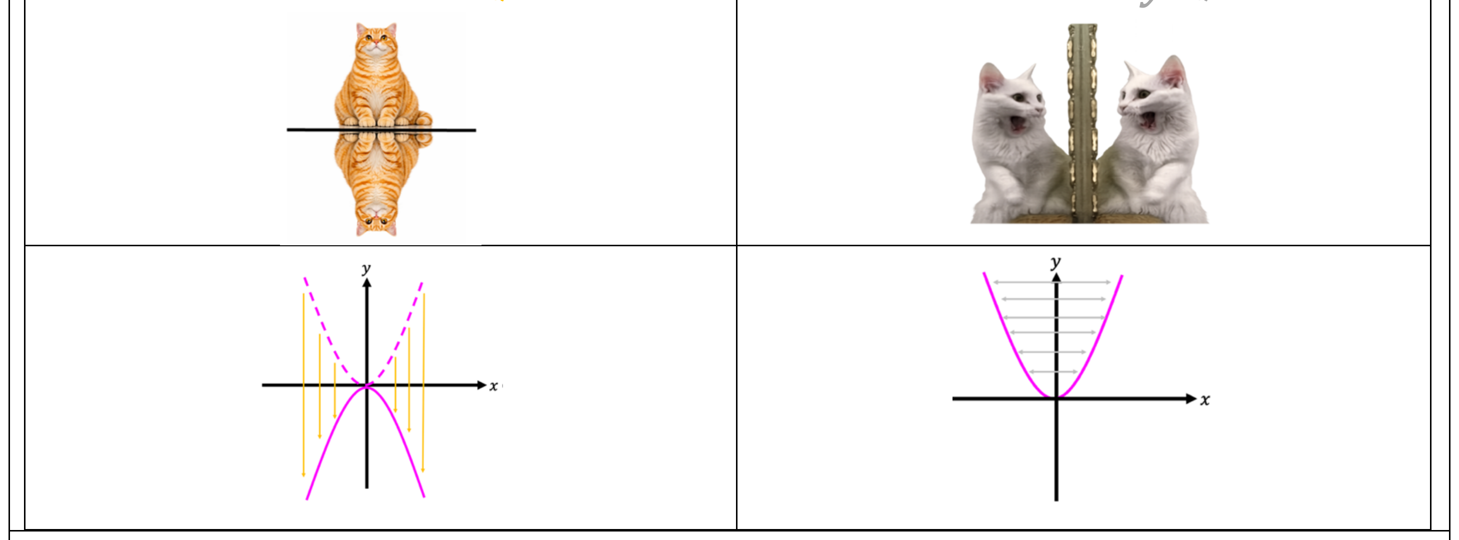
Enlargements (stretch or shrink by a scale factors)

Make It Fatter | Make It Thinner | Make It Taller | Make It Shorter



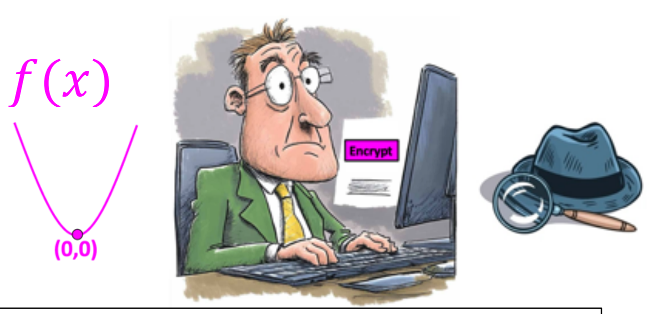
Reflections

Reflect it in x axis | Reflect it in y axis



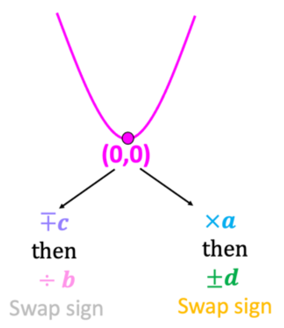
Easy to understand right?

Unfortunately, transformations can be written in a more formal way. Rather than write the transformations in words, we write them in an **encrypted way** with what they do to the curve $f(x)$



becomes

$af(x)$
$f(bx)$
$f(x \pm c)$
$f(x) \pm d$
$-f(x)$
$f(-x)$



$\mp c$ then $\div b$ Swap sign

$\times a$ then $\pm d$ Swap sign

This will be explained more in column 3 of this cheat sheet using your function knowledge later on

Key:

- a: stretches in y direction (multiply every y value by a)
  - a bigger than 1 makes TALLER
  - a less than 1 makes SHORTER)
- b: stretches in x direction (divide every x value by b)
  - b bigger than 1 makes thinner
  - b less than 1 makes fatter
- c= moves c units in x direction
  - $-c$ adds moves to right i.e. add c to x
  - $+c$ moves to left i.e. subtract c from x
- d= moves d units in y direction
  - $-d$ moves down i.e. subtract d from y
  - $+d$ moves to up i.e. add d to y
- $-$: reflection in y axis (multiply every y value by -1)
- $-$: reflection in x axis (multiply every x value by -1)

can write c and d as a translation vector $\begin{pmatrix}\mp c\\ \pm d\end{pmatrix}$

How can we remember what all of these do?

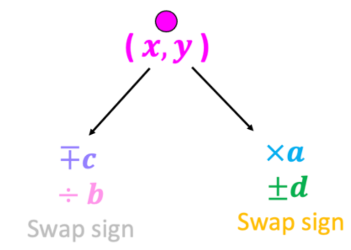
" Anything inside a bracket affects x (and does the opposite you expect) and anything outside a bracket affects y"

x does the opposite to BIDMAS and y obeys BIDMAS

Basic Examples – General Function Form

" Anything inside a bracket affects x (and does the opposite you expect) and anything outside a bracket affects y"

$af(x)$
$f(bx)$
$f(x \pm c)$
$f(x) \pm d$
$-f(x)$
$f(-x)$

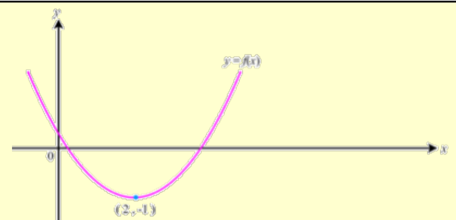


$\mp c$ $\div b$ Swap sign

$\times a$ $\pm d$ Swap sign

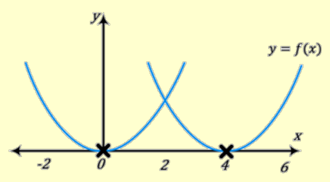
The diagram shows a sketch of $f(x)$. State what will happen the turning point under the following transformations

i. $f(x+2)$
ii. $3f(x)$
iii. $f(2x)$
iv. $f\left(\frac{1}{3}x\right)$
v. $f(x)+4$
vi. $f(-x)$

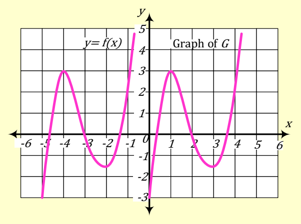


$f(x+2)$	$3f(x)$	$f(2x)$	$f\left(\frac{1}{3}x\right)$	$f(x)+4$	$f(-x)$
We start with $f(x)$ $f(x+2)$ We have a number inside a bracket. Anything inside a bracket affects x and does the opposite to what we expect. We expect to add 2, but instead we subtract 2 from every x coordinate (we don't change the y coordinate since this s not a y transformation) $(2,-1)$ becomes $(2-2,-1)$ Simplifies to $(0,-1)$	We start with $f(x)$. $3f(x)$ We have a number outside a bracket. This affects y and does the same as what we expect). We expect to multiply by 3 so we do exactly this to every y coordinate (we don't change the x coordinate since this s not an x transformation) $(2,-1)$ becomes $(2,-1(3))$ Simplifies to $(2,-3)$	We start with $f(x)$. $f(2x)$ We have a number inside the bracket. Anything inside a bracket affects x and does the opposite to what we expect). We expect to multiply by 2 but instead we divide every x coordinate by 2 (we don't change the y coordinate since this s not a y transformation) $(2,-1)$ becomes $\left(\frac{2}{2},-1\right)$ Simplifies to $(1,-1)$	We start with $f(x)$. $f\left(\frac{1}{3}x\right)$ We have a number inside the bracket. Anything inside a bracket affects x and does the opposite to what we expect). We expect to divide by 3 but instead we multiply every x by 3 (we don't change the y coordinate since this s not a y transformation) $(2,-1)$ becomes $(2(3),-1)$ Simplifies to $(6,-1)$	We start with $f(x)$. $f(x)+4$ We have a number outside the bracket. This affects y and does the same as we expect. We expect to add 4 so we do exactly this to every y coordinate (we don't change the x coordinate since this s not an x transformation) $(2,-1)$ becomes $(2,-1+4)$ Simplifies to $(2,3)$	We start with $f(x)$. $f(-x)$ The negative inside the bracket means a reflection in the y axis (it swaps the sign of the x value). $(2,-1)$ becomes $(-2,-1)$

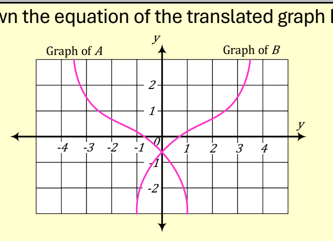
The curve with equation $y = f(x)$ is translated so that the point $(0,0)$ is mapped onto the point $(4,0)$. Find an equation of the translated curve.



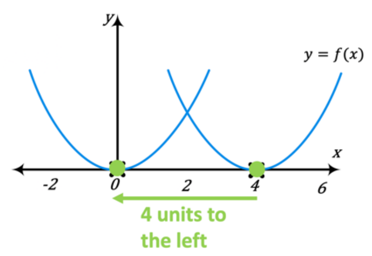
The graph of G is a translation of the graph of y= $f(x)$. Write down in terms of f, the equation of the graph G



Write down the equation of the translated graph B



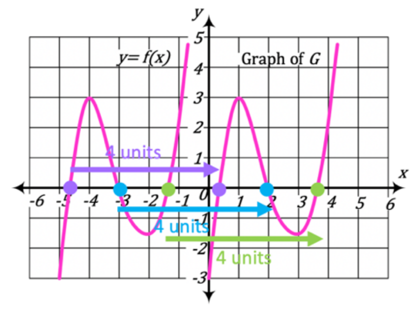
Let's pick the green turning point to compare.



The graph of $y = f(x)$ has been moved 4 units in the negative x direction (to the left) and hence we do the opposite inside the bracket

$f(x+4)$

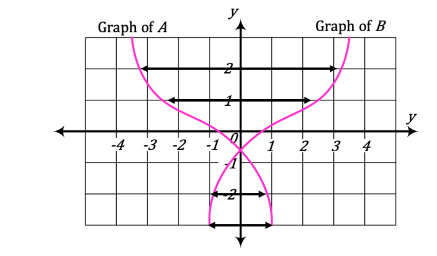
Let's pick some corresponding points (the colour pairs below)



The graph of f has been moved 5 units in the positive x direction to obtain the graph of g

$G(x) = f(x-5)$

Let's have a look at the graphs. They cross therefore it is a reflection. Let's have a look at some correspending points.

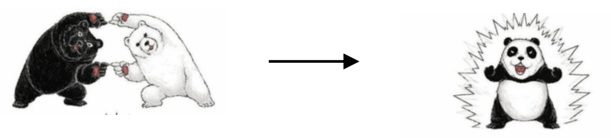


The transformation is a refection in the y axis (so the transformation is affecting the x values).

Let Graph of A be $f(x)$ then Graph of B is $-f(x)$.

Combined Transformations

We are sometimes not just given single transformations. We may be given more than one, so let's see what all transformationswould look like writtne together

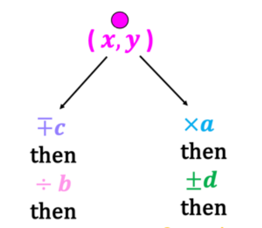


$f(x) \rightarrow$ $af(x)$, $f(bx)$, $f(x \pm c)$, $f(x) \pm d$, $-f(x)$, $f(-x)$ $\rightarrow -af(-bx \pm c) \pm d$

We can do the x ones first or the y ones, but when we do each the **order matters**

Important

c before b and a before d

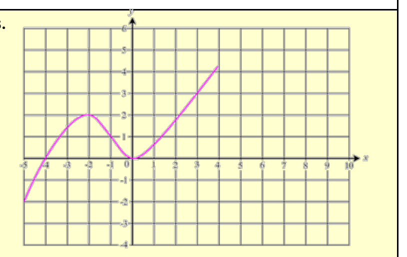


$\mp c$ then $\div b$ then Swap sign

$\times a$ then $\pm d$ then Swap sign

The diagram shows a sketch of $f(x)$. State what happens to the points $(0,0), (-2,2), (-4,0)$ under each of the following transformations.

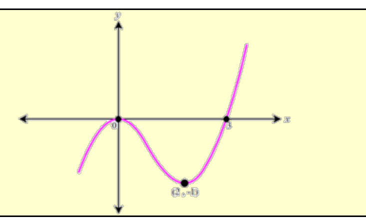
i. $f(x-2)+3$
ii $-f(-2x)$
iii. $3f\left(\frac{1}{2}x\right)-1$
iv. $2f(2x-1)+1$



i.	ii.	iii.	iii.
$f(x-2)+3$ • x transformations: add 2 • y transformations: add 3 $(0,0) \rightarrow (0+2, 0+3) = (2,3)$ $(-2,2) \rightarrow (-2+2, 2+3) = (0,5)$ $(-4,0) \rightarrow (-4+2, 0+3) = (-2,3)$	$-f(-2x)$ • x transformations: divide by -2 • y transformations: multiply by -1 $(0,0) \rightarrow \left(\frac{0}{-2}, (-1)(0)\right) = (0,0)$ $(-2,2) \rightarrow \left(\frac{-2}{-2}, (-1)(2)\right) = (-1,-2)$ $(-4,0) \rightarrow \left(\frac{-4}{-2}, (-1)(0)\right) = (2,0)$	$3f\left(\frac{1}{2}x\right)-1$ • x transformations: divide by $\frac{1}{2}$ means multiply by 2 • y transformations: multiply by 3 THEN subtract 1 $(0,0) \rightarrow (2(0), (3)(0)-1) = (0,0)$ $(-2,2) \rightarrow (2(-2), (3)(2)-1) = (-4,5)$ $(-4,0) \rightarrow (2(-4), (3)(0)-1) = (-8,-1)$	$2f(2x-1)+1$ • x transformations: add 1 THEN divide by 2 • y transformations: multiply by 2 THEN add 1 $(0,0) \rightarrow \left(\frac{0+1}{2}, 2(0)+1\right) = (0.5,1)$ $(-2,2) \rightarrow \left(\frac{-2+1}{2}, 2(2)+1\right) = (-0.5,5)$ $(-4,0) \rightarrow \left(\frac{-4+1}{2}, 2(0)+1\right) = (-\frac{3}{2},1)$

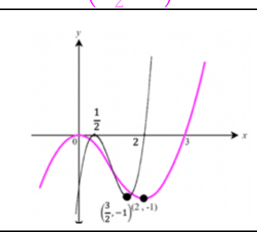
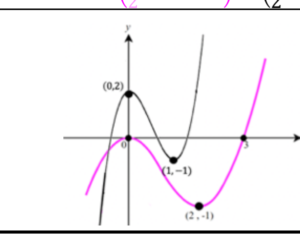
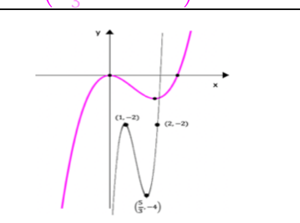
The graph shows a sketch of $f(x)$. On another grid draw the following graphs

i. $f(2x-1)$
ii. $3f(2x)+2$
iii. $2f(3x-3)-1$



Let's consider a few important points of this graph and apply the transformations. In this case it is be the local minimum, local maximum and the root of the graph:

$(0,0)$, $(2,-1)$ and. $(3,0)$

i.	ii.	iii.Fix graph
$(0,0) \rightarrow \left(\frac{0+1}{2}, 0\right) \rightarrow \left(\frac{1}{2}, 0\right)$ $(2,-1) \rightarrow \left(\frac{2+1}{2}, -1\right) \rightarrow \left(\frac{3}{2}, -1\right)$ $(3,0) \rightarrow \left(\frac{3+1}{2}, 0\right) \rightarrow (2,0)$	$(0,0) \rightarrow \left(\frac{0}{2}, 3(0)+2\right) \rightarrow (0,2)$ $(2,-1) \rightarrow \left(\frac{2}{2}, 3(-1)+2\right) \rightarrow (1,-1)$ $(3,0) \rightarrow \left(\frac{3}{2}, 3(0)+2\right) \rightarrow \left(\frac{3}{2}, 2\right)$	$(0,0) \rightarrow \left(\frac{0+3}{3}, 2(0)-1\right) \rightarrow (1,-1)$ $(2,-1) \rightarrow \left(\frac{2+3}{3}, 2(-1)-1\right) \rightarrow (\frac{5}{3},-3)$ $(3,0) \rightarrow \left(\frac{3+3}{3}, 2(0)-1\right) \rightarrow (2,-1)$
		

Quadratics In Completed The Square Form

Consider

$f(x) = x^2$

Think back to when you first learnt the topic functions.

$f(x) = x^2$
$f(2) = 2^2 = 4$
$f(x) + 2 = x^2 + 2$

The same knowledge uses for this applies with transformations

$af(x)$	$f(bx)$	$f(x \pm c)$	$f(x) \pm d$	$-f(x)$	$f(-x)$
the entire function is multiplied by a $= ax^2$	replace every x in the function by bx $(bx)^2$	replace every x in the function by $x \pm c$ $(x \pm c)^2$	add $\pm d$ to the entire function $x^2 \pm d$	multiply the entire function by -1 $-x^2$	replace every x in the function by $-x$ $(-x)^2$

Hence we have the following

$f(x) = x^2$
$af(x) = ax^2$
$f(bx) = (bx)^2$
$f(x \pm c) = (x \pm c)^2$
$f(x) \pm d = x^2 \pm d$
$-f(x) = -x^2$
$f(-x) = (-x)^2$

ax^2 means multiply y by a

$(bx)^2$ means divide x by b (opposite to what we expect)

$(x+c)^2$ means subtracts units from x hence move left (opposite to what we expect)

$(x-c)^2$ means add c units to x hence move right minus (opposite to what we expect)

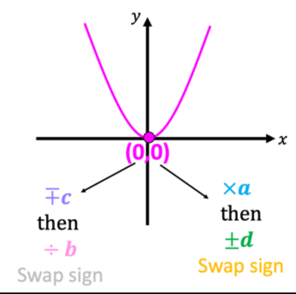
$x^2 + d$ means add d units to y hence move up (the same as what we expect)

$x^2 - d$ means subtracts d units from y hence move down (the same as what we expect)

$-x^2$ means reflection in x axis hence multiplies y by -1 (swaps the sign of the y coordinate)

Writing these all in one go

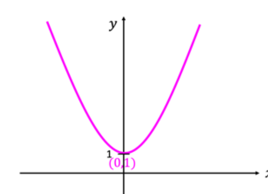
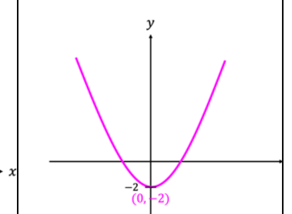
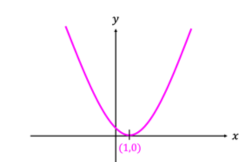
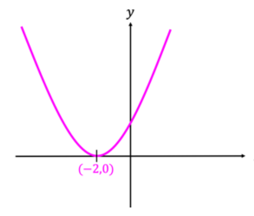
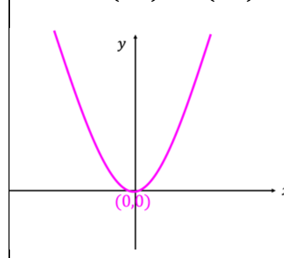
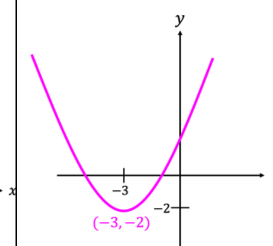
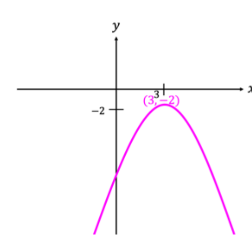
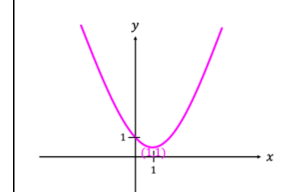
$$x^2 \rightarrow -a(-bx \pm c)^2 \pm d$$



$\mp c$ then $\div b$ Swap sign

$\times a$ then $\pm d$ Swap sign

Basic Graphing Examples

Graph $y = x^2 + 1$	Graph $y = x^2 - 2$	Graph $y = (x-1)^2$	Graph $y = (x+2)^2$
$y = x^2 + 1$ Add 1 to y Hence $(0,0) \rightarrow (0,1)$ 	$y = x^2 - 2$ Subtract 2 from y Hence $(0,0) \rightarrow (0,-2)$ 	$y = (x-1)^2$ Add 1 to x Hence $(0,0) \rightarrow (1,0)$ 	$y = (x+2)^2$ Subtract 2 from x Hence $(0,0) \rightarrow (-2,0)$ 
Graph $y = 3x^2$	**Graph $y = (x+3)^2 - 2$**	**Graph $y = -(x-3)^2 - 2$**	**Graph $y = 2(x-1)^2 + 1$**
$y = 3x^2$ Multiply y by 3 which doesn't change anything since $0 \times$ anything =0 Hence $(0,0) \rightarrow (0,0)$ 	$y = (x+3)^2 - 2$ Subtract 3 from x and subtract 2 from y Hence $(0,0) \rightarrow (-3,-2)$ 	$y = -(x-3)^2 - 2$ add 3 to x, reflect in x axis and subtract 2 from y Hence $(0,0) \rightarrow (3,-2)$ 	$y = 2(x-1)^2 + 1$ add 1 to x, multiply y by 2 and add 1 to y Hence $(0,0) \rightarrow (1,1)$ 

Let $f(x) = 3(x+1)^2 - 12$
Let $g(x) = x^2$. The graph of f may be obtained from the graph of g by 2 transformations.

- A stretch of scale factor t in the y direction
  Followed by
- A translation of $\begin{pmatrix}p\\ q\end{pmatrix}$

Find $\begin{pmatrix}p\\ q\end{pmatrix}$ and the value of t

Consider x^2

Let's colour code to see the transformations

$3(x+1)^2 - 12$

- A stretch of scale factor t in the y direction
  This is 3
- A horizontal translations of p units
  This is -1 (1 unit to the left) since we do the opposite
- A vertical translation of q units
  This is -12

The last 2 transformations can be combined as a translation of $\begin{pmatrix}p\\ q\end{pmatrix} = \begin{pmatrix}-1\\ -12\end{pmatrix}$

$t = 3$

$\begin{pmatrix}-1\\ -12\end{pmatrix}$

The graph of $y = x^2$ may be transformed into the graph of $y = 5 - 3(x-4)^2$ by these transformations

- A reflection in the line $y = 0$
- A vertical stretch with scale factor k
- A horizontal translations of p units
- A vertical translation of q units

Write down the value of k, p and q

We must first complete the square as we can't see/spot the transformations for quadratics without it. Completing the square is not covered in detail in this sheet. See my completing the square notes, cheat sheets or work sheet by topic for this

Completing the square on $y = 2x^2 - 10x + 21$ gives

$$y = -2\left(x - \frac{5}{2}\right)^2 + \frac{17}{2}$$

$$y = -2\left(x - \frac{5}{2}\right)^2 + \frac{17}{2}$$

- A vertical stretch with scale factor k
- A horizontal translation of p units
- A vertical translation of q units

$k = 2$
$p = 2.5$
$q = \frac{17}{2}$

Note: The reflection hasn't been asked for here

The quadratic function f is defined by
$3x^2 - 15x + 13$
i.Write f in the form
$f(x) = 3(x-h)^2 - k$
The graph of f is translated 4 units in the positive x direction and 6 units in the positive y direction.
ii.Find the function g for the translated graph, giving your answer in the form $ax^2 + bx + c$

i.
We must first complete the square to be able to spot the transformations

$f(x) = 3(x - 2.5)^2 - \frac{23}{4}$

ii.
Translated 4 units in the positive x direction (recall that we do the opposite for x transformations and hence subtract 4) and translated 6 units in the positive y direction.

$$g(x) = 3\left(x - \frac{5}{2} - 4\right)^2 - \frac{23}{4} + 6$$

Simplify

$$g(x) = 3\left(x - \frac{13}{2}\right)^2 + \frac{1}{4}$$

Expand the bracket and collect like terms to get the form required

$$g(x) = 3x^2 - 39x + 127$$

The graph of $y = x^2$ is transformed into the graph of
$y = 2x^2 - 10x + 21$ by the following transformations:
A vertical stretch with scale factor k
followed by
a horizontal translation of p units
followed by
a vertical translation of q units.
Write down the values of k, p and q

We must first complete the square as we can't see/spot the transformations for quadratics without it. Completing the square is not covered in detail in this sheet. See my completing the square notes, cheat sheets or work sheet by topic for this

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$$y = -2\left(x - \frac{5}{2}\right)^2 + \frac{17}{2}$$

- A vertical stretch with scale factor k
- A horizontal translation of p units
- A vertical translation of q units

$k = 2$
$p = 2.5$
$q = \frac{17}{2}$

Note: The reflection hasn't been asked for here

Quadratics In General form

Consider

$$f(x) = ax^2 + bx + c$$

Think of your function knowledge. How do the transformations already mentioned appear in the function?

$$ex^2 + fx + g \rightarrow \quad e(bx \pm c)^2 + f(bx \pm c) + g + d$$

How do the transformations already mentioned appear in the function?

$f(bx)$ means replace every x in the function by bx

$f(x \pm c)$ means replace every x in the function by $x \pm c$

$f(x) \pm d$ means add $\pm d$ to the entire function

Hence

$$ax^2$$
$$f(bx) = (bx)^2$$
$$f(x \pm c) = (x \pm c)^2$$
$$f(x) \pm d = x^2 \pm d$$
$$-f(x) = -x^2$$
$$f(-x) = (-x)^2$$

*****If the question looks really bad to spot just complete the square on both, graph and compare what has happened to the vertex to spot the transformation*****

$f(x \pm c)$

To move c spaces to the left we add c and to move c spaces to the right we subtract c, BUT we must add c wherever x appears in the function

$(x-2)^3 - 6(x-2)^2 + 9(x-2)$ is the graph of $x^3 - 6x^2 + 9$ moved 2 units to the right.

This should make sense with your function knowledge since $f(x+c)$ tells us to substitute $(x+c)$ for EVERY x value that we see.

$f(bx)$

To divide all x values by b we must multiply by b wherever x appears in the function

$g(x) = (2x)^3 - 6(2x)^2 + 9(2x)$

is the graph of $f(x) = x^3 - 6x^2 + 9$ shrunk by a scale factor of 2 in the x direction.

$f(x) \pm d$

To add d we must add d to the function

$g(x) = -2x^2 + 8x + 10$ is the graph of

$f(x) = -2x^2 + 8x + 5$ moved 5 units up

How?

$$g(x) = -2x^2 + 8x + 10 + 5$$

We can see that 5 has been added to y and the x terms have not changed.
The whole equation is y that is why by tagging $+5$ on at the end we are still affecting EVERY y.

$f(x) = -2x^2 + 8x + 5$ has been translated to obtain the graph of :

i. $g(x) = -2(x+3)^2 + 8(x+3) + 5$.

List the transformations.

ii. $h(x) = -2(x-1)^2 + 8(x-1) + 8$. List the transformations.

How does $f(x) = -2x^2 + 8x + 5$ become

$$g(x) = -2(x+3)^2 + 8(x+3) + 5?$$

$$g(x) = -2(x+3)^2 + 8(x+3) + 5$$

We can see that 3 has been added to EVERY x (which we need in this form) and nothing else has changed

We do the opposite and subtract 3 since x transformations do the opposite

Note:

We had to subtract 3 from EVERY x value since every x gets affected

The transformation is: move 3 units to the left. We can write this as translation $\begin{pmatrix}-3\\0\end{pmatrix}$

ii.

Let's do way 1 above as it is quicker

How does $f(x) = -2x^2 + 8x + 5$ become

$$h(x) = -2(x-1)^2 + 8(x-1) + 8?$$

$$g(x) = -2(x-1)^2 + 8(x-1) + 5 + 3$$

We can see that 1 has been subtracted from EVERY x (which we need in this form)

We do the opposite and add 1 since x transformations do the opposite

We can also see that 3 has been added to every y (since the whole equation is equal to y so by adding 3 all y's are affected)

The transformation is: move 1 unit to the right and 3 units up. We can write this as translation $\begin{pmatrix}1\\3\end{pmatrix}$

$f(x) = -2x^2 + 8x + 5$ has been translated to obtain the graph of $g(x) = -2x^2 + 8x + 10$.

List the transformations.

$f(x) = -2x^2 + 8x + 5$ becomes

$$g(x) = -2x^2 + 8x + 10$$

How?

$$g(x) = -2x^2 + 8x + 5 + 5$$

We can see that 5 has been added to y and the x term hasn't changed.

Note:

The whole equation is y that is why by tagging +5 on at the end we are affecting every y.

The transformation is: move 5 units upwards. We can write this as translation $\begin{pmatrix}0\\5\end{pmatrix}$

The curve with equation $f(x) = 5x^2 + 9x + 2$ is transformed to the curve with equation $g(x) = 5(x+4)^2 + 9(x+4) + 8$ by the translation $\begin{pmatrix}a\\b\end{pmatrix}$. Write down the value of a and the value of b

How does $f(x) = 5x^2 + 9x + 2$ become

$$g(x) = 5(x+4)^2 + 9(x+4) + 8?$$

$$g(x) = 5(x+4)^2 + 9(x+4) + 2 + 6$$

We can see that 4 has been added to EVERY x (which we need in this form)

We do the opposite and subtract 4 since x transformations do the opposite

We can also see that 6 has been added to every y (since the whole equation is equal to y so by adding 6 all y's are affected)

The transformation is: move 4 unit to the left and 6 units up

We can write this as translation $\begin{pmatrix}-4\\6\end{pmatrix}$

A curve L has equation $y = x^2 + 7x + 20$. The curve L is transformed to another curve S under the translation $\begin{pmatrix}2\\0\end{pmatrix}$

Find an equation for S. Give your answer in the form $y = ax^2 + bx + c$

$$y = x^2 + 7x + 20$$

This is the same as writing

$$f(x) = x^2 + 7x + 20$$

$\begin{pmatrix}2\\0\end{pmatrix}$ tells us we have the transformation $f(x-2)$

$$f(x-2) = (x-2)^2 + 7(x-2) + 20$$

$$f(x-2) = x^2 - 4x + 4 + 7x - 14 + 20$$

$$f(x-2) = x^2 + 3x + 10$$

The quadratic function f is defined by $3x^2 - 15x + 13$

i. Write f in the form $f(x) = 3(x-h)^2 - k$

The graph of f is translated 4 units in the positive x direction and 6 units in the positive y direction.

ii. Find the function g for the translated graph, giving your answer in the form $ax^2 + bx + c$

i.

We must first complete the square as we can't see/spot the transformations or apply transformations to $3x^2 - 15x + 13$ that easily
(well you can, but it requires a bit more thought. I will show both ways below anyway).

Completing the square is not covered in detail in this sheet. See my completing the square notes, cheat sheets or work sheet by topic for this

Completing the square on $3x^2 - 15x + 13$ gives $3\left(x - \frac{5}{2}\right)^2 - \frac{23}{4}$

ii.

Way 1: Using the completed the square form (best way since have already done this in a previous part of the question)

Now we have the correct form to analyse

Translated 4 units in the positive x direction (recall that we do the opposite for x transformations and hence subtract 4) and translated 6 units in the positive y direction.

.

$$g(x) = 3\left(x - \frac{5}{2} - 4\right)^2 - \frac{23}{4} + 6$$

Simplify

$$g(x) = 3\left(x - \frac{13}{2}\right)^2 + \frac{1}{4}$$

Expand the bracket and collect like terms to get the form required

$$g(x) = 3x^2 - 39x + 127$$

Way 2: Without Completing The Square

$$3x^2 - 15x + 13$$

Translated 4 units in the positive x direction (recall that we do the opposite for x transformations and hence subtract 4) and translated 6 units in the positive y direction.

$$g(x) = 3(x-4)^2 - 15(x-4) + 13 + 6$$

Notice how we add 4 to every x value since every x gets affected
The whole equation is y that is why tagging +6 on at the end we are affecting every y

$$g(x) = 3(x^2 - 8x + 16) - 15x + 60 + 13 + 6$$

$$g(x) = 3x^2 - 24x + 48 - 15x + 60 + 13 + 6$$

$$g(x) = 3x^2 - 39x + 127$$

$f(x) = -2x^2 + 8x + 5$ has been translated to obtain the graph of

$g(x) = -2(x+4)^2 + 8x + 22$. List the transformations.

Since 4 has not been added to EVERY x this is a bit harder to spot without completing the square.

How does $f(x) = -2x^2 + 8x + 5$ become

$$g(x) = -2(x+4)^2 + 8x + 22?$$

We need every x to change in the same way for it to be an x transformation. We can force the second x to be $(x+4)$ and then undo the extra term we create

$$f(x) = -2(x+4)^2 + 8(x+4) + 22 - 32$$
$$f(x) = -2(x+4)^2 + 8(x+4) - 10$$

We started with $f(x) = -2x^2 + 8x + 5$ and want to get $g(x) = -2(x+4)^2 + 8(x+4) - 10$

$$f(x) = -2(x+4)^2 + 8(x+4) + 5 - 15$$

We can see that 4 has been added to x (however, we do the opposite since x transformations do the opposite hence subtract 4) and 15 has been subtracted from y

Note: we could have complete the square on both functions

$f(x) = x^2 - 2x + 3$ has been translated to obtain the graph of
$g(x) = x^2 - 4x + 2$. List the transformations.

We have no choice but to complete the square for both functions here

$f(x) = x^2 - 2x + 3$	$g(x) = x^2 - 4x + 3$
Completing the square gives $f(x) = (x-1)^2 + 2$	Completing the square gives $g(x) = (x-2)^2 - 1$

How does $f(x) = (x-1)^2 + 2$ becomes

$$g(x) = (x-2)^2 - 1$$

$$g(x) = (x - 1 - 1)^2 + 2 - 3$$

We can see that 1 has been subtracted from x (we do the opposite and add 1) and 3 has been subtracted from y

The transformation: Move 1 unit to the right and 3 units down. We can write this as translation $\begin{pmatrix}1\\-3\end{pmatrix}$

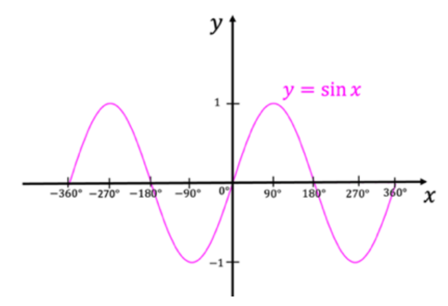
Trig Equations

To graph trig graphs we always use transformations: Locate the coordinates of the original graph

$$\sin x$$
$$\cos x$$
$$\tan x$$

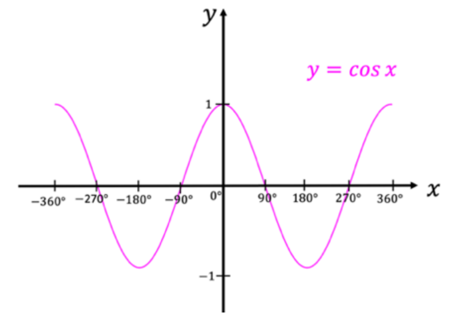
$$\sin x \rightarrow -a\,\sin(-bx \pm c) \pm d$$
$$\cos x \rightarrow -a\,\cos(-bx \pm c) \pm d$$
$$\tan x \rightarrow -a\,\tan(-bx \pm c) \pm d$$

$y = \sin x$



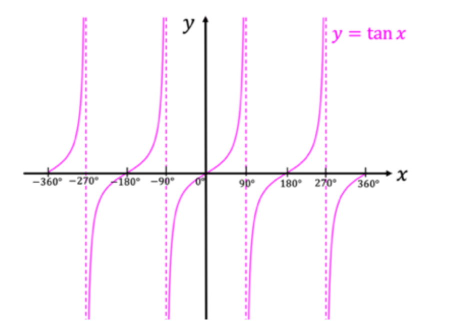
Apply the transformations to the points
$(0°, 0), (90°, 1), (180°, 0), (270°, -1), (360°, 1)$
$(-90°, 1), (-180°, 0), (-270°, -1), (-360°, 1)$

$y = \cos x$



Apply the transformations to the points
$(0°, 1), (90°, 0), (180°, -1), (270°, 0), (360°, 1)$
$(-90°, 0), (-180°, -1), (-270°, 0), (360°, 1)$

$y = \tan x$



Apply the transformations to the points
$(0°, 0), (180°, 0), (360°, 0), (-180°, 0), (-360°, 1)$

And asymptotes
$x = 90°, x = 270°, x = -90°, x = -270°$

Describe the geometrical transformation that maps the graph of $y = \sin x$ onto the graph of

$$y = \sin(5x + 10).$$

Consider $y = \sin x$

$$y = \sin(5x + 10)°$$

x transformations (remember to reverse BIDMAS since we do the opposite to what we expect):

- Move 10 degrees to the left (subtract 10° to every x)
- A stretch scale factor 5 parallel to the y axis (divide every x by 5)

Describe the geometrical transformation that maps the graph of

$f(x) = \sin x$ onto the graph of $g(x) = \sin(2x - 1) + 3$

Consider $f(x) = \sin x$

$$g(x) = \sin(2x - 1) + 3$$

x transformations (remember to reverse BIDMAS since we do the opposite to what we expect):

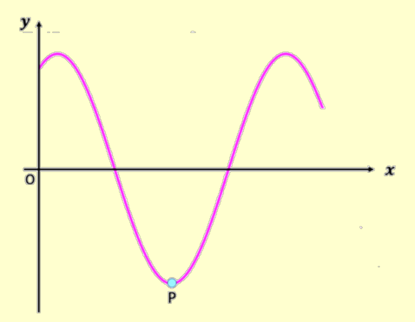
- Move 1 unit to the right (add 1 to every x)
- A stretch scale factor $\frac{1}{2}$ parallel to the y axis (divide every x by 2)

y transformations:

- Move 3 units up (add 3 to every y)

Note: The x and y transformations can be done in any order, meaning we can do the y transformations before the x transformation or vice versa.

The graph below shows part of the curve with equation $y = 5\cos(x - 30)°, x \geq 0$. The point P on the curve is the minimum point with the smallest positive x coordinate.



State the coordinates of P

Let's use transformations

The min of the graph of original $y = \cos x$ is $(180°, -1)$

Here we have $y = 5\cos(x - 30)°$

x transformation:

- Move 30° to the right (add 30° to every x value)

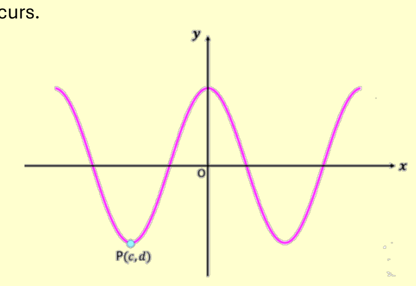
y transformation:

- A stretch scale factor 5 parallel to the y axis (multiply every y by 5)

$(180°, -1)$ becomes $(180° + 30°, -1(5))$

=(210,−5)

The graph below shows part of the curve with equation $y = 3\cos x°$. The point (c, d) is a minimum point on the curve with c being the smallest negative value of x at which a minimum occurs.



i. State the value of c and the value of d

State the coordinates of the point to which P is mapped by the transformation which transforms the curve with equation $y = 3\cos x°$ to the curve with equation

ii. $y = 3\cos\left(\frac{x°}{4}\right)$

iii. $y = 3\cos(x - 36)°$

i.

Let's use transformations

The neg min of the graph of original $y = \cos x$ is $(-180°, -1)$

Here we have $y = 3\cos(x)°$

This transformation is a stretch scale factor 3 parallel to the y axis (multiply every y by 3)

$(-180°, -1)$ becomes $(-180°, -1(3))$

$=(-180°, -3)$

$$c = -180$$
$$d = -3$$

ii.

Here we have $y = 3\cos\left(\frac{x}{4}\right)° = 3\cos\left(\frac{1}{4}x\right)°$

This transformation is a stretch scale factor 3 parallel to the y axis (multiply every y by 3)
This transformation is a stretch scale factor 4 parallel to the x axis (multiply every x by 4)

$(-180°, -1)$ becomes $(-180°(4), -1(3))$

$=(-720°, -3)$

iii.

Here we have $y = 3\cos(x - 36)°$

This transformation is a stretch scale factor 3 parallel to the y axis (multiply every y by 3)
Move 36° to the right (add 30° to every x value)

$(-180°, -1)$ becomes $(-180° + 36, -1(3))$

$=(-144°, -3)$